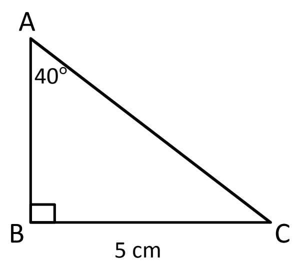
FOM 11 **3.1 Exploring Side-Angle Relationships in Acute Triangles**

In Grade 10 you learned how to solve for sides and angles in a right triangle using

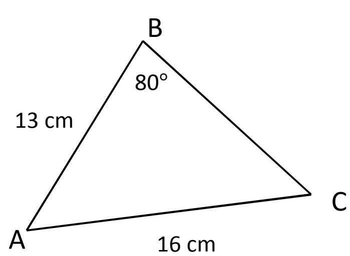
SOH CAH TOA.

Example 1: Solve the triangle:

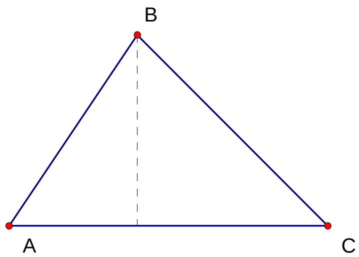


We can use SOH CAH TOA to solve for sides and angles in acute triangles as well.
Acute triangles are triangles where all angles are less than 90°.

Example 2: Solve for $\angle A$.



Example 3: Find two equivalent expressions that represent the height of ΔABC.

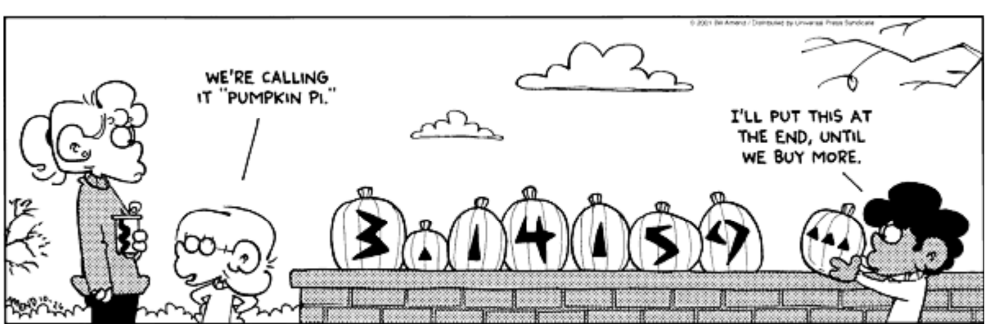


This shows that the ratios of $\dfrac{\text{length of opposite side}}{\sin(\mathit{angle})}$ are equivalent for all side-angle pairs in an acute triangle.

ie. $$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

Assignment:

pg. 117 #1-5



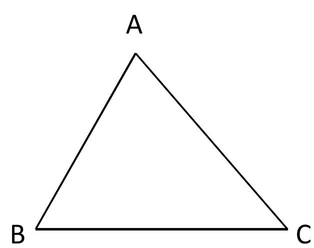
3.2 Proving and Applying The Sine Law

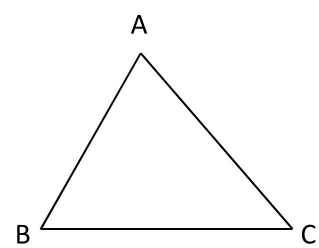
The Sine Law states:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

OR $$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$

Proof:

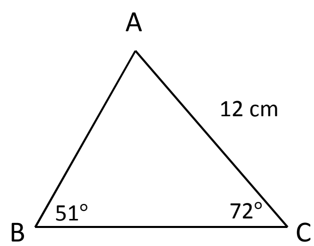




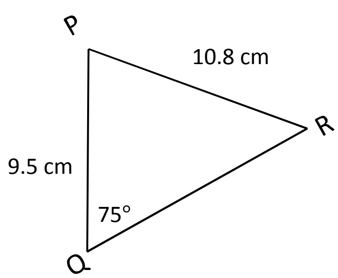
To use the Sine Law we need either:

- Two angles and one side.
- Two sides and an uncontained angle.

Example 1: Solve the triangle:



Example 2: Solve the triangle:



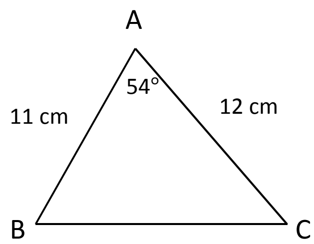
Example 3: The course for a boat race starts at point A, and heads in a direction S40°W to point B, then in a direction S54°E to point C, and finally north back to point A. The distance from A to C is 12 km. Find the total distance of the boat race.

Assignment: pg. 124 #1-5, 7-10, 12, 17

FOM 11 **3.3 Proving and Applying The Cosine Law**

The Sine Law cannot always help you determine unknown sides and angles in an acute triangle.

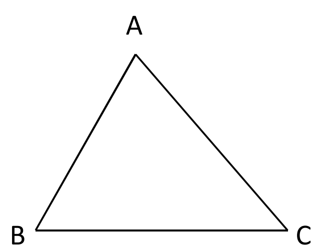
Eg.



Therefore, another relationship is needed, called the **Cosine Law**.

The Cosine Law states: $$a^2 = b^2 + c^2 - 2bc\cos A$$.

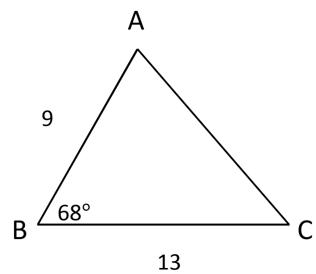
Proof:



To use the Cosine Law we need either:

- Two sides and the contained angle.
- All three sides.

Example 1: Solve the triangle:



Example 2: Solve ΔABC where a = 5, b = 7, and c = 10.

Example 3: The distance from home plate to center field at a baseball stadium is 400 ft. What is the angle at center field between the lines of sight to short stop (half way between 2^{nd} and 3^{rd} base), and home plate? Reminder: In a baseball diamond, there is 90 ft between each adjacent base.

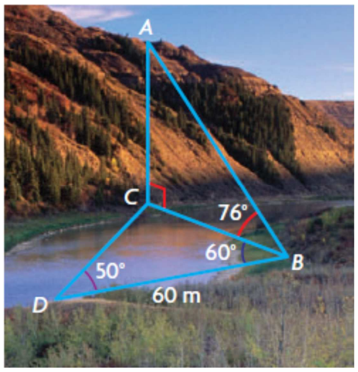
Assignment: pg. 137 #2-5, 6ac, 7b, 8, 9, 11, 13

3.4 Solving Problems Using Acute Triangles

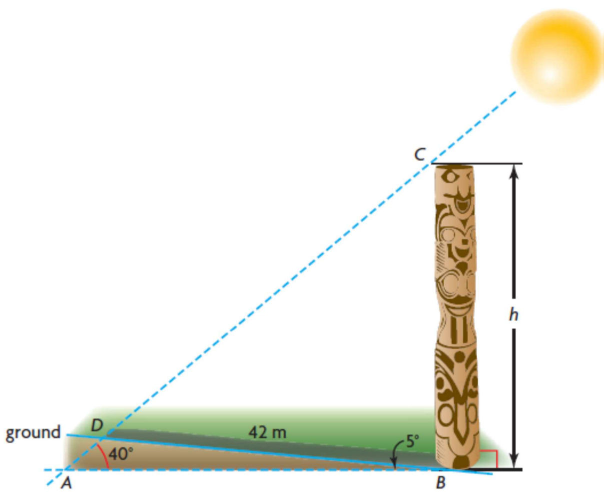
Example 1: When a plane is coming in to land at a 2510-m runway, the angles of depression to the ends of the runway are 10° and 13° respectively. How far is the plane from the end of the runway?

Example 2: Brendan and Diana plan to climb the cliff at Dry Island Buffalo Jump, Alberta. They need to know the height of the climb before they start. Brendan stands at point *B*, as shown in the diagram. He uses a clinometer to determine $\angle ABC$, the angle of elevation to the top of the cliff. Then he estimates $\angle CBD$, the angle between the base of the cliff, himself, and Diana, who is standing at point *D*. Diana estimates $\angle CDB$, the angle between the base of the cliff, herself, and Brendan.

Determine the height of the cliff to the nearest metre.



Example 3: The world's tallest free-standing totem pole is located in Beacon Hill Park in Victoria, British Columbia. It was carved from a single cedar log by noted carver Chief Mungo Martin of the Kwakiutl (Kwakwaka'wakw), with a team that included his son David and Henry Hunt. It was erected in 1956. While visiting the park, Manuel wanted to determine the height of the totem pole, so he drew a sketch and made some measurements:



• I walked along the shadow of the totem pole and counted 42 paces, estimating each pace was about 1 m.
• I estimated that the **angle of elevation** of the Sun was about 40°.
• I observed that the shadow ran uphill, and I estimated that the angle the hill made with the horizontal was about 5°.

How can Manuel determine the height of the totem pole to the nearest metre?

Assignment: pg. 147 #3-6, 8, 11-14