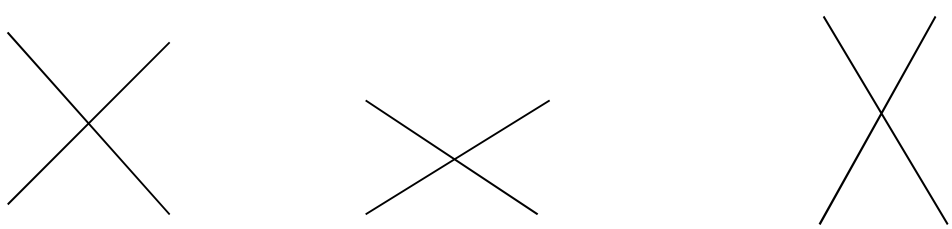
FOM 11 **1.1 Making Conjectures: Inductive Reasoning**

If the same result occurs over and over again, we may conclude that it will always occur. This kind of reasoning is called **inductive reasoning**.

Inductive reasoning can lead to a **conjecture**, which is a testable expression that is based on available evidence but is not yet proved.

Example 1: Use inductive reasoning to make a conjecture about the product of an odd integer and an even integer.

Example 2: Make a conjecture about intersecting lines and the angles formed.



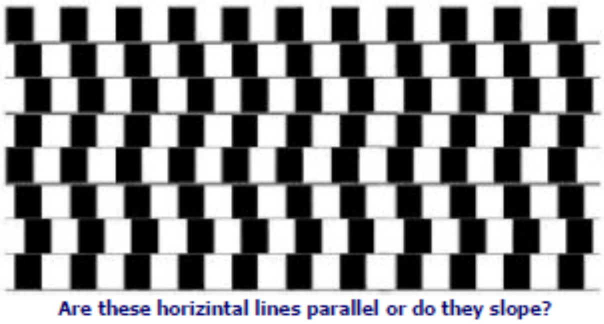
Example 3: Make a conjecture about the sum of two odd numbers.

Assignment: pg. 12 #3, 5, 6, 9, 10-12, 14, 16, 20

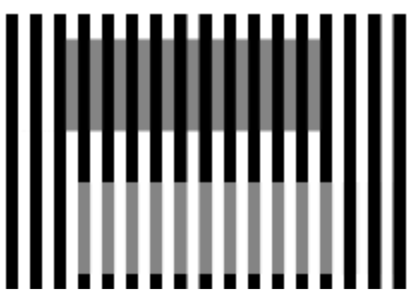
FOM 11 **1.2 Exploring the Validity Of Conjectures**

Some conjectures initially seem to be valid, but are shown not to be valid after more evidence is gathered.

Example 1: Make a conjecture about the lines below:



Example 2: Make a conjecture about the grey rectangles:



The best we can say about a conjecture reached through inductive reasoning is that there is **evidence either to support or deny it**.

Assignment: pg. 17 #1-3

….. An Introduction to Sudoku

2	1	5	4	6	3
3	6	4	2	5	1
6	4	3	5	1	2
1	5	2	3	4	6
5	2	6	1	3	4
4	3	1	6	2	5

<u>Goal:</u> Fill each 2 x 3 box with the digits 1 to 6. Place them in such a way that the numbers 1 to 6 only appear **<u>once</u>** in **each** **<u>row</u>** and **<u>column</u>**

	5		1		
5		3	4		2
6		2	5		3
		4		2	

In the more difficult puzzles there are many occasions when two or more possible numbers can go in each cell and you will usually have to pencil these possibilities in using small writing before deciding which goes where

Before this stage there are always some cells that can have only one solution. It is better to try and find all these solutions first, before moving on to find solutions for cells that have multiple possible solutions.

In this 2x3 box there is only one place to put the 5. Look along the rows and columns to see why.

	5		1		
5		3	4		2
6		2	5		3
		4		2	

Use the same method to place a 5 in every 2 by 3 box. (Remember you can only have one 5 in each row and column

	5		1		
5		3	4		2
6		2	5		3
		4		2	
		5			

Use the same method to place all the numbers that only have one solution. Think logically, start from 1 and try to place it in every 2x3 box. Then move through all the numbers placing only the ones you know for certain will fit.

5	6	1	4	9	7	8	3	2
8	3	2	6	5	1	4	7	9
7	9	4	8	3	2	5	1	6
6	1	8	5	4	9	7	2	3
4	7	9	1	2	3	6	5	8
2	5	3	7	6	8	1	9	4
3	4	7	2	8	5	9	6	1
1	2	6	9	7	4	3	8	5
9	8	5	3	1	6	2	4	7

Sudoku

….Now The Real Thing!

The object now is to place the digits 1 to 9 in each 3 x 3 box in such a way that the numbers 1 to 9 only appear once in each row and column of the large 9 x 9 grid.

As in the smaller puzzles there may be occasions when two or more possible numbers can go in each cell. Always try to fill in all the cells that only have one possible solution first!

In this 3x3 square there is only one place to put the 4. Look along the rows and columns to see why.

	3		6		9		8	
		4						9
9	5			8			6	
4		6	1			2		
7	1	5	3		6	8	9	4
		9			8	6		7
	7			1			4	6
5						9		
	9		8		5		7	

Use the same method to place a 4 in every 3x3 square

	3		6		9		8	
		4						9
9	5			8			6	
4		6	1			2		
7	1	5	3		6	8	9	4
		9			8	6		7
	7			1			4	6
5	4					9		
	9		8		5		7	

- Use the same method to place all the numbers that only have **one solution**. Think logically, start from 1 and try to place it in every 3x3 square. Then move through all the numbers placing only the ones you know for certain will fit.
- Go through all Row's and Columns to see if there are any cell's with only **one solution.**
- Go through it all again….maybe you have eliminated some of the multiple solutions.
- Continue on in this manner until you have filled in all the boxes. If you become stuck use little numbers in the top of the box like this 4,7 to show which numbers could go there. (This will come in handy in more difficult puzzles.)

FOM 11 **1.3 Using Reasoning to Find A Counterexample to a Conjecture**

We know that inductive reasoning can lead to a conjecture, which may or may not be true. One way a conjecture may be proven false is by a **counterexample**.

Example 1: If possible, find a counterexample for each conjecture. If not, write “true”.

a. Conjecture: Every mammal has fur.

b. Conjecture: The acute angles in a right triangle are equal.

c. Conjecture: A polygon has more sides than diagonals.

d. Conjecture: The square of every even number is even.

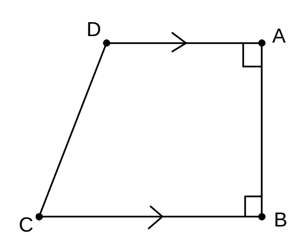
e. Conjecture: An even number is any number which is not odd.

Example 2: Three conjectures are given.
For which conjectures is this diagram a counterexample?

A. The opposite sides of a parallelogram are equal.

B. A quadrilateral cannot have both a 90° angle and an obtuse angle.

C. Every trapezoid has 2 pairs of equal angles.



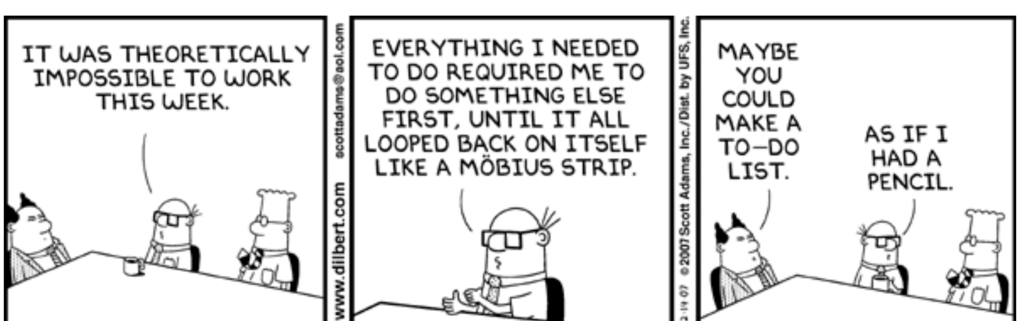
Assignment: pg. 22 #1, 3-6, 10, 12, 14, 17

FOM 11 **1.4 Proving Conjectures: Deductive Reasoning**

When we make a conclusion based on statements that we accept as true,
we are using **deductive reasoning**.

Example 1: Use deductive reasoning to prove that the product of an odd integer and an even integer is even.

Example 2: Use deductive reasoning to prove that opposite angles of intersecting lines are equal.



Example 3: Use deductive reasoning to prove that the difference between consecutive perfect squares is always an odd number.

Example 4: Weight-lifting builds muscle. Muscle makes you strong. Strength improves balance. Inez lifts weights. What can be deduced about Inez?

Assignment: pg. 31 #1, 2, 4-7, 10, 11, 15, 19

FOM 11 **1.4.2 Deductive Reasoning Part II**

When we make a conclusion based on statements that we accept as true, we are using **deductive reasoning**. The rules we follow when performing algebraic manipulations are things that we accept (and know) as true. So we are using deductive reasoning to prove a statement is always true.

Statements that we know are true:

Any integer multiplied by 2 is an even number.

- This means that $2x$ or 2(any combination of variables and coefficients) will **always** be even.

If you add 1 to any even integer you will get an odd number.

- This means that $2x + 1$ or 2(any combination of variables and coefficients) + 1 will **always** be odd.

Consecutive Numbers follow each other in numerical order

- This means that x, $x + 1$, $x + 2$, $x + 3$ are 4 numbers that come one after the other numerically.
- $2x$, $2x + 2$, $2x + 4$, $2x + 6$ are 4 consecutive even numbers
- $2x + 1$, $2x + 3$, $2x + 5$, $2x + 7$ are 4 consecutive odd numbers

Example 1: Use deductive reasoning to prove that the sum of an odd number and an even number is always odd.

Finishing a Proof:

- If proving an answer is even it should look like this → 2(any combination of variable terms)
- If proving an answer is odd it should look like this → 2(any combination of variable terms) + 1
- If proving an answer is divisible by 3 it should look like this → 3(any combination of variable terms)
- If proving an answer is divisible by 4 it should look like this → 4(any combination of variable terms)
- If proving an answer is divisible by 5 it should look like this → 5(any combination of variable terms)
- etc……

Example 2: Prove that the square of an even integer is always even

Example 3: Prove that the result of the number trick below is always the number you start with.

- Choose a number
- Add 2
- Multiply by 3
- Subtract 6
- Subtract your original number
- Divide by 2

Example 4: The sum of a two digit number and its reversal is a multiple of 11.

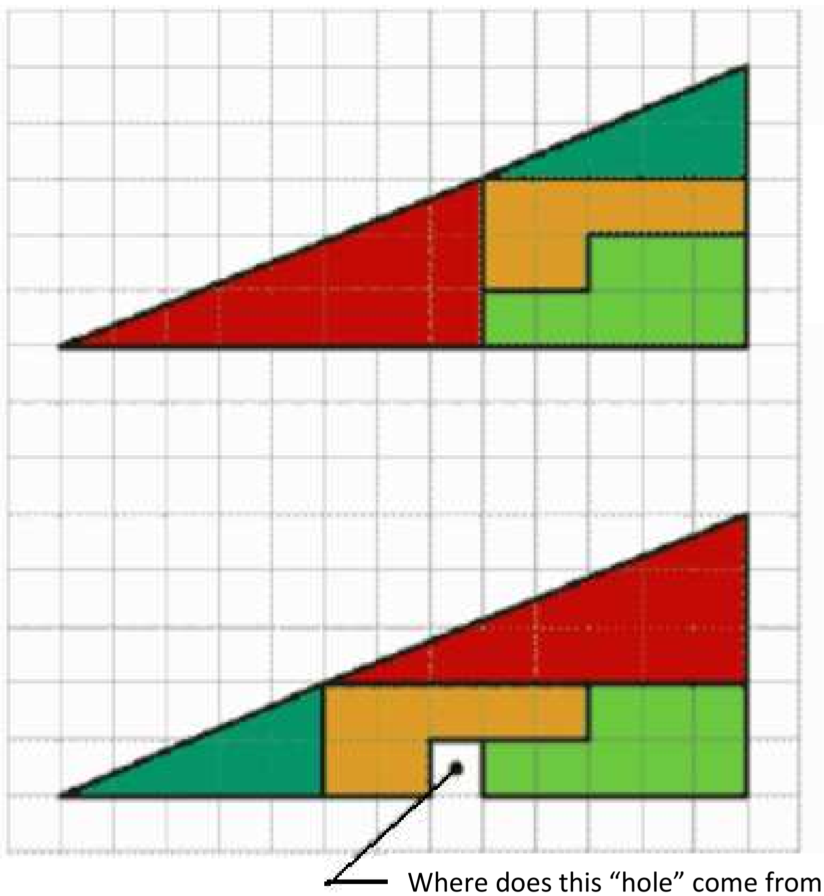
Assignment: Deductive Reasoning Worksheet

FOM 11 **1.5 Proofs That Are Not Valid**

A single error in a deductive proof will make it invalid. Some common errors are:

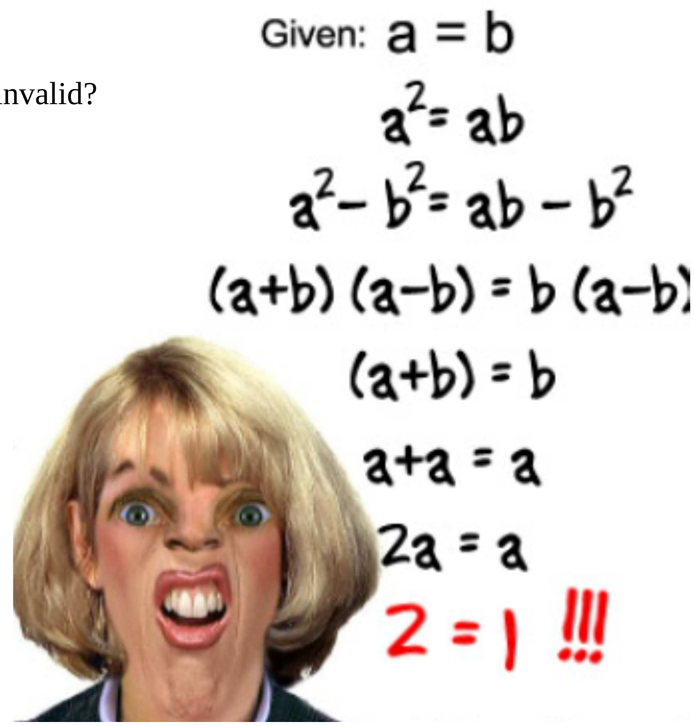
- Dividing by zero.
- Circular reasoning.
- Confusing reasoning.

Example 1:



Example 2:

Why is this proof invalid?



Example 3: Isaac claims that -3 = 3.

Proof: Assume -3 = 3.

$$(-3)^2 = 3^2$$

$$9 = 9$$

Therefore: -3 = 3.

Where did Isaac go wrong?

Assignment: pg. 42 #1, 3, 5, 6, 7, 10

FOM 11 **1.6 Reasoning to Solve Problems**

Reminder: A **conjecture** is a conclusion based on examples.

We know that **inductive reasoning** can lead to a conjecture that may be proven by **deductive reasoning**. However, conjectures may be false, and can be disproven by a **counterexample**.

Example 1: Decide whether the process used is inductive or deductive reasoning:

a. Show the sum of two even numbers is even by using several examples.

b. No mathematician is boring. Ann is a mathematician. Therefore, Ann is not boring.

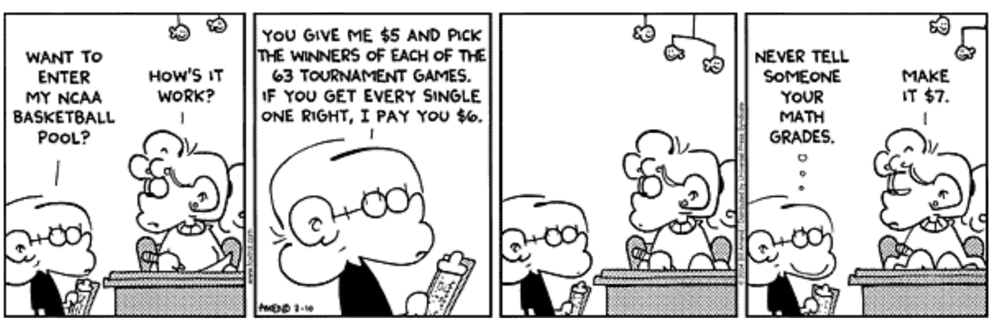
c. One counterexample proves that a conjecture is false.

d. You show why your statement makes sense.

e. You give evidence that your statement is true.

f. Six other examples to show that your conjecture is true.

g. What three coins have a value of $0.60?



Example 2: Al, Bob, Cal, and Dave are on four sports teams.

- Each play on just one team.
- They play football, basketball, baseball, and hockey.
- Bob is a goalie.
- The tallest player plays basketball, and the shortest baseball.
- Cal is taller than Dave, but shorter than Al and Bob.

What sports does each play?

Example 3: Art, Bill, Cecil, and Don live in the same apartment. They are a manager, teacher, artist and musician. Art and Cecil watch TV with the teacher. Bill and Don go to the hockey game with the manager. Cecil jogs with the manager and teacher. Who is the manager?

Assignment: pg. 48 #1, 3, 5, 6, 8, 9, 10, 13, 16

PROBLEM SOLVING: LOGIC PUZZLES

FOM 11 **Notes**

A Logic Puzzle is a word problem which requires the use of Mathematical Deductive Reasoning to solve. Deductive Reasoning, is the process of working from one or more general statements to reach a logically **certain** conclusion.

Example 1: The Boxes

There are three boxes. One is labeled "APPLES" another is labeled "ORANGES". The last one is labeled "APPLES AND ORANGES". You know that each is **labeled incorrectly**. You may ask me to pick one fruit from one box which you choose. How can you label the boxes correctly?

Example 2: Mary's mum has four children.
The first child is called April.
The second May.
The third June.
What is the name of the fourth child?

Often Logic Puzzles include clues to help you find the solution.

If a logic puzzle seems too difficult, it is often helpful to use a table to keep track of the clues.

Example 3: Danny is having a birthday party with 6 of his family members. They are his grandmother, mother, aunt, brother, father, and uncle. Their names in random order are Ben, Lily, Jeff, Betty, Jane, and Luke. Look at the clues to discover the names of Danny's family members.

	Ben	**Lily**	**Jeff**	**Betty**	**Jane**	**Luke**
Grandmother						
Mother						
Aunt						
Brother						
Father						
Uncle						

CLUES:

1. Ben is not Danny’s uncle.
2. Danny’s grandmother’s name starts with B.
3. Luke is not Danny’s brother.
4. Lily is not his aunt.
5. Danny’s father’s name is Jeff.

Example 4: Three little pigs, who each lived in a different type of house, handed out treats for Halloween. Use the clues to figure out which pig lived in each house, and what type of treat each pig handed out.

CLUES:

1. Petey Pig did not hand out popcorn.
2. Pippin Pig does not live in the wood house.
3. The pig that lives in the straw house, handed out popcorn.
4. Petunia Pig handed out apples.
5. The pig who handed out chocolate, does not live in the brick house.

	Straw	Wood	Brick	Popcorn	Apples	Chocolate
Petey						
Pippin						
Petunia						
Popcorn						
Apples						
Chocolates						

Drawing Pictures often helps to sort out clues.

Example 5: Alex, Bret, Chris, Derek, Eddie, Fred, Greg, Harold, and John are nine students who live in a three storey building, with three rooms on each floor. A room in the West wing, one in the centre, and one in the East wing. If you look directly at the building, the left side is West and the right side is East. Each student is assigned exactly one room. Can you find where each of their rooms is:

CLUES:

1. Harold does not live on the bottom floor.
2. Fred lives directly above John and directly next to Bret (who lives in the West wing).
3. Eddie lives in the East wing and one floor higher than Fred.
4. Derek lives directly above Fred.
5. Greg lives directly above Chris.

	West Wing	Centre	East Wing
3^{rd} floor			
2^{nd} floor			
1^{st} floor			

Example 7: (The Big One)

During a recent music festival, four DJs entered the mixing contest. Each wore a number, either 1, 2, 3 or 4 and their decks were different colours. Can you determine who came where, which number they wore and the colour of their deck?

CLUES:

1. DJ Skinf Lint came first, and only one DJ wore the same number as the position he finished in.
2. DJ Slam Dunk wore number 1.
3. The DJ who wore number 2 had a red deck and DJ Jam Jar didn't have a yellow deck.
4. The DJ who came last had a blue deck.
5. DJ Park'n Ride beat DJ Slam Dunk.
6. The DJ who wore number 1 had a green deck and the DJ who came second wore number 3.

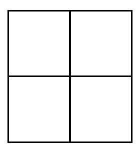
	First	Second	Third	Fourth	Red Deck	Yellow Deck	Blue Deck	Green Deck	1	2	3	4
DJ Skinf Lint												
DJ Slam Dunk												
DJ Jam Jar												
DJ Park’n Ride												
1												
2												
3												
4												
Red Deck												
Yellow Deck												
Blue Deck												
Green Deck												

FOM 11 **1.7 Analyzing Puzzles And Games**

Both inductive and deductive reasoning are useful for determining a strategy to solve a puzzle or win a game.

Example 1: Use four 9's in a math equation that equals 100.

Example 2: The following figure is made up of 12 sticks. Can you move just two sticks and create seven squares?



Example 3: Put the numbers 1 to 8 in each square so that each side adds to the middle term.

	12	

	13	

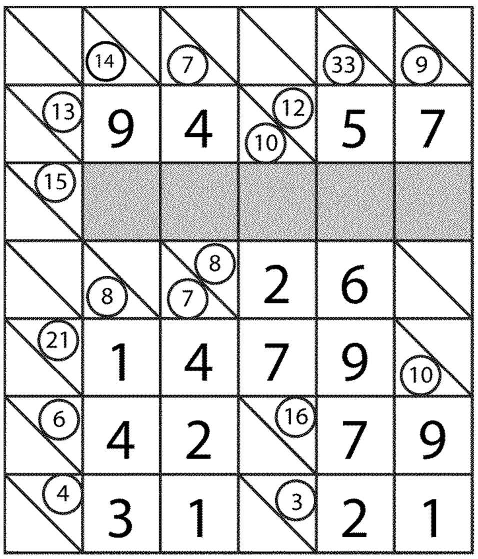
	14	

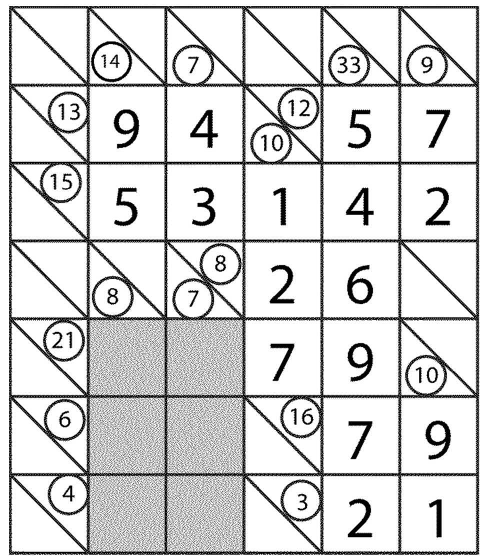
	15	

Kakuro is an arithmetic puzzle in a grid. You must place the digits 1 to 9 into a grid of squares so that **each horizontal or vertical run of white squares adds up to the clue** printed either to the left of or above the run.

No digit can be repeated within any single run. Runs end when you reach a non-white square. Every puzzle has **a single unique solution** and can be solved purely by logic - **no guessing is required**.

Example 4: Complete the following Kakuro puzzles by filling in the grey squares.





Assignment: pg. 55 #4, 5, 6, 7, 9, 10, 11