

Until now, you have used the primary trigonometric ratios only with acute angles. Here we will investigate the values of the primary trigonometric ratios for **obtuse angles**.

$\theta$	$\sin\theta$	$\cos\theta$	$\tan\theta$	$(180^\circ - \theta)$	$\sin(180^\circ - \theta)$	$\cos(180^\circ - \theta)$	$\tan(180^\circ - \theta)$
$90^\circ$							
$100^\circ$							
$110^\circ$							
$120^\circ$							
$130^\circ$							
$140^\circ$							
$150^\circ$							
$160^\circ$							
$170^\circ$							
$180^\circ$							

What relationships do you observe when comparing the trigonometric ratios for obtuse angles with the trigonometric ratios for the related supplementary acute angle?

**Example 1:** Calculate each ratio to 4 decimal places. Predict another angle that will have an **opposite trigonometric ratio**. Check your Prediction:

$$\cos 127^\circ$$

$$\tan 102^\circ$$

$$\sin 166^\circ$$

$$\cos 145^\circ$$

$$\sin 71^\circ$$

$$\tan 38^\circ$$

**Example 2:** Calculate each ratio to 4 decimal places. Predict another angle that will have an **equal trigonometric ratio**. Check your Prediction:

$$\tan 136^\circ$$

$$\cos 155^\circ$$

$$\sin 23^\circ$$

$$\cos 94^\circ$$

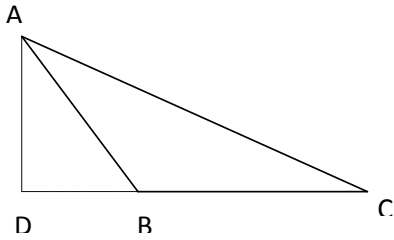
$$\tan 140^\circ$$

$$\sin 68^\circ$$

**FOM 11      4.2 Proving and Applying the Sine and Cosine Laws for Obtuse Triangles**

The Sine Law works for **obtuse triangles** as well as acute triangles.

Proof:

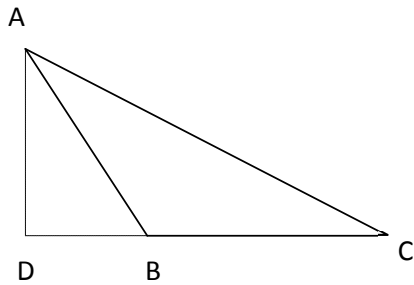


**Example 1:** Solve  $\triangle ABC$ , given  $\angle B = 29^\circ$ ,  $\angle C = 105^\circ$ , and  $b = 30$  cm.

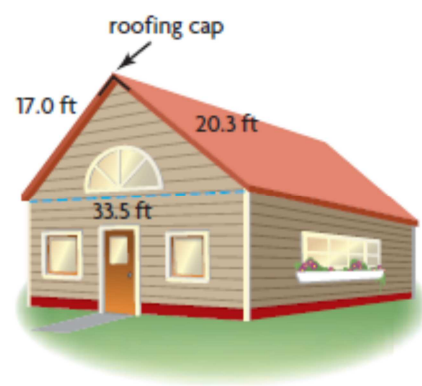
**Example 2:** Given  $\triangle ABC$ , with  $\angle A = 120^\circ$ ,  $a = 12$ , and  $b = 10$ , find  $\angle B$ .

The Cosine Law also holds true for obtuse triangles.

Proof:



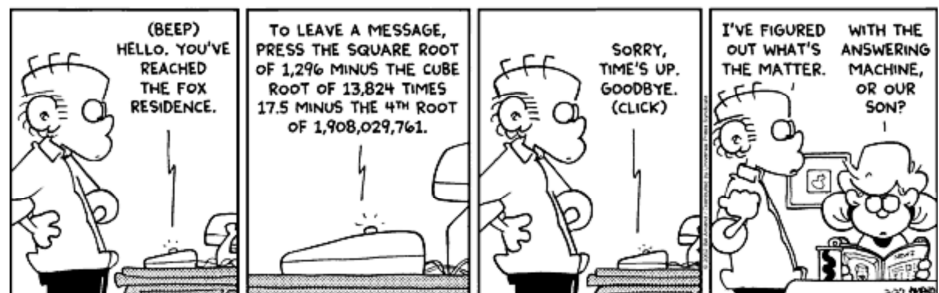
**Example 3:** The roof of a house consists of two slanted sections, as shown. A roofing cap is being made to fit the crown of the roof, where the two slanted sections meet. Determine the measure of the angle of elevation for each roof section, to the nearest tenth of a degree.



**Example 4:** A plane flies 820 km from A to B at a bearing of  $N68^\circ E$ . Then it flies 600 km from B to C with a bearing of  $N40^\circ E$ . Find the distance from C to A.

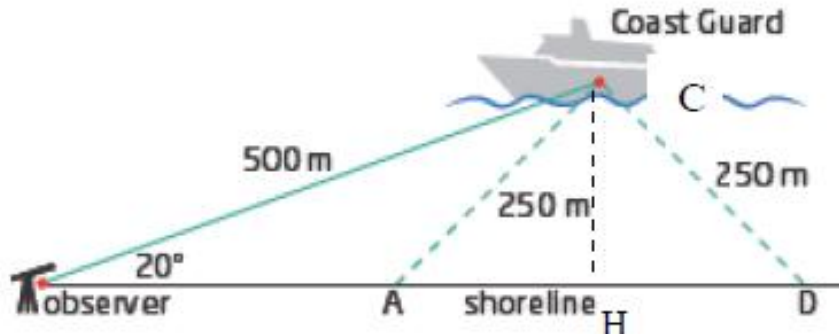
**Example 5:** A clock has two hands that are 12 cm and 15 cm long. What is the distance, to the nearest tenth of a centimetre, between the tips of the hands at 5 p.m.?

Assignment:  
pg. 170 #1, 2, 4, 6-9,  
12-14

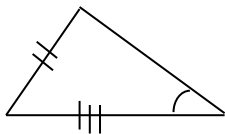


**Try This Question:**

The Canadian Coast Guard Pacific Region is responsible for more than 27 000 km of coastline. The rotating spotlight from the Coast Guard ship can illuminate up to a distance of 250 m. An observer on the shore is 500 m from the ship. His line of sight to the ship makes a  $20^\circ$  with the shoreline. What length of shoreline is illuminated by the spotlight



When we are given two sides and an angle opposite one of the sides, we may be dealing with the **ambiguous case**.

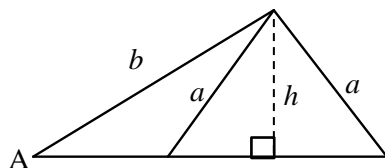


ASS (angle-side-side) or SSA (side-side-angle)

**The Ambiguous Case** occurs when the side opposite the angle is longer than the height, but shorter than the other side.

In this picture,  $a$  is the opposite side from the given angle. It is shorter than side  $b$  (the other side) but longer than the height of the triangle ( $h$ ).

$$h < a < b$$



We know that  $\sin\theta$  and  $\sin(180 - \theta)$  are equal. So without changing any of the “SSA” measures given. We can see that the angle opposite  $b$  can be acute or obtuse. This creates two possible triangles, and two possible measures for the third side.

**Example 1:** Given each SSA situation for  $\triangle ABC$ , determine if the information leads to the ambiguous case (determine if two triangles are possible)

a.  $\angle A = 30^\circ, a = 6, b = 10.$

b.  $\angle A = 62^\circ, a = 11, b = 10.$

c.  $\angle A = 48^\circ, a = 12, b = 21.$

d.  $\angle A = 50^\circ, a = 15, b = 17.$

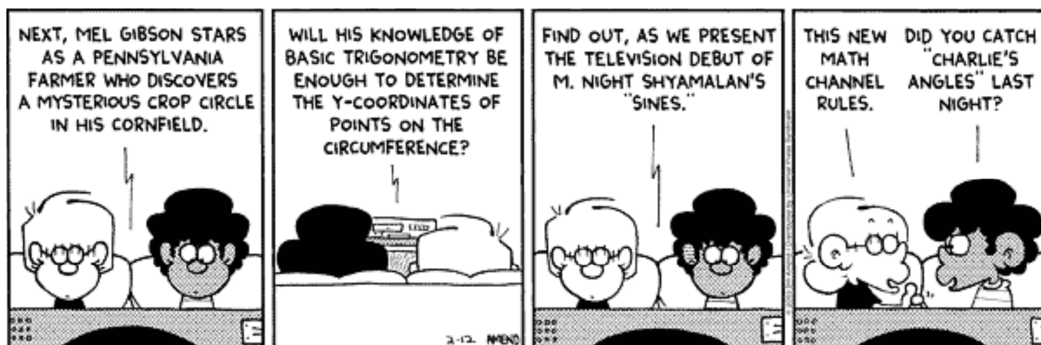
**Example 2:** Martina and Carl are part of a team that is studying weather patterns. The team is about to launch a weather balloon to collect data. Martina's rope is 7.8 m long and makes an angle of  $36^\circ$  with the ground. Carl's rope is 5.9 m long. Assuming that Martina and Carl form a triangle in a vertical plane with the weather balloon, what is the distance between Martina and Carl, to the nearest tenth of a metre?



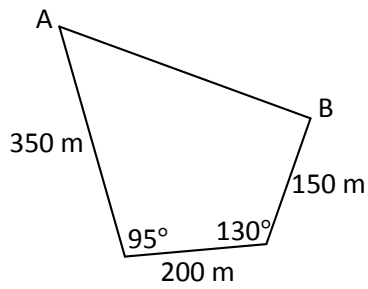
**Example 3:** Leanne and Kerry are hiking in the mountains. They left Leanne's car in the parking lot and walked northwest for 12.4 km to a campsite. Then they turned due south and walked another 7.0 km to a glacier lake. The weather was taking a turn for the worse, so they decided to plot a course directly back to the parking lot. Kerry remembered, from the map in the parking lot, that the angle between the path to the campsite and the path to the glacier lake measures about  $30^\circ$ . How long do they have to walk before they reach the parking lot?

**Example 1:** Three circles of radius 3, 5, and 7 cm are tangent to each other. Find the largest angle formed by joining their centers.

**Example 2:** A plane is sighted by two observers 1 km apart at angles  $74^\circ$  and  $78^\circ$ . The observers and the plane are in the same vertical plane. How high is the plane?



**Example 3:** An irregular plot of land has dimensions as shown. Find AB.



**Example 4:** From the top of a 30 m observation tower, a fire ranger observes smoke at a bearing of  $90^\circ$  with an angle of depression of  $5^\circ$ . The ranger spots more smoke at a bearing of  $200^\circ$  with an angle of depression of  $2^\circ$ . How far apart are the sources of smoke (to the nearest metre)?