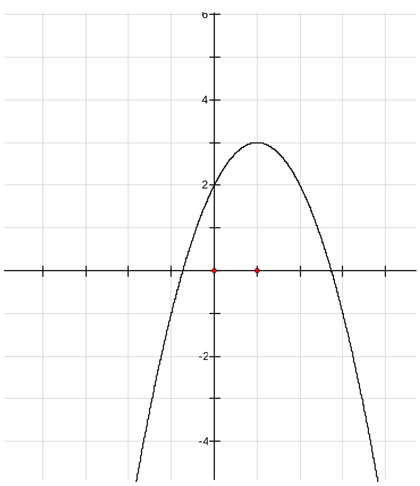
FOM 11 **7.1 Exploring Quadratic Relations**

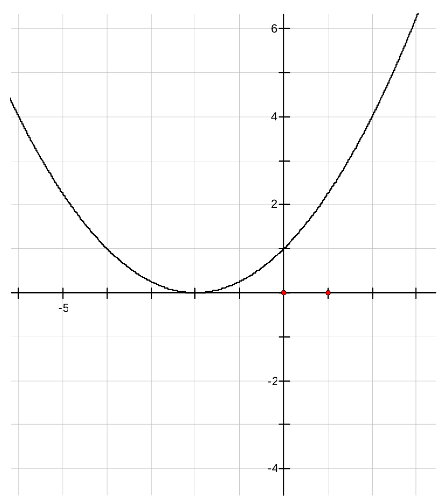
A ***quadratic function*** is a function of degree 2 (ie. The highest exponent on the variable is 2). The equation of a quadratic function can be written in the form $y = ax^2 + bx + c$, where *a, b,* and *c* are constants, and $a \neq 0$. Some examples are:

$$y = x^2 \qquad f(n) = n^2 - 4 \qquad y = 3x^2 - 4x - 7$$

The graph of every quadratic function is called a **parabola**.

Examples:





Properties of a parabola:

- **Vertex**: the maximum or minimum point on the graph
- **Y-intercept:** where the graph crosses the y-axis (ie. Where x = 0)
- **X-intercept(s)**: where the graph crosses the x-axis (ie. Where y = 0)
- **Axis of symmetry**: the line about which the parabola is reflected. It goes through the vertex. If the vertex is located at point (*m*, *n*), the equation of the axis of symmetry is $x = m$.
- **Domain:** the set of valid x-values for the graph.
- **Range:** the set of valid y-values for the graph.

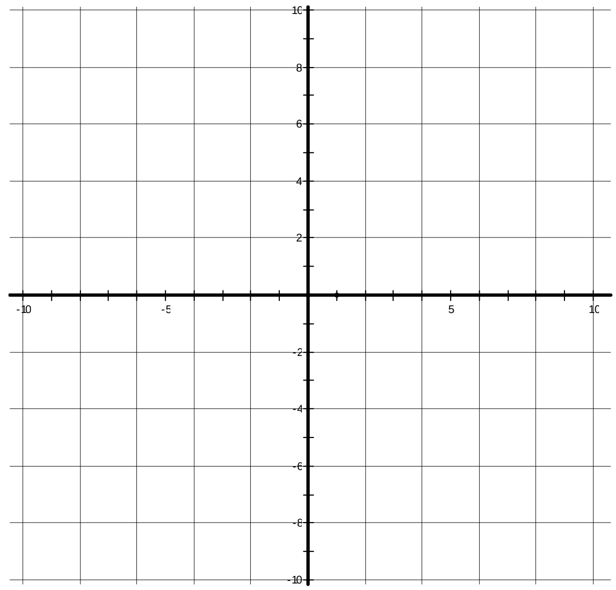
Example 1: Graph the following quadratic functions on the same grid:

a. $y = x^2 + 2x - 3$

b. $y = -2x^2 + 2x - 3$

c. $y = -\frac{1}{2}x^2 + 2x - 3$

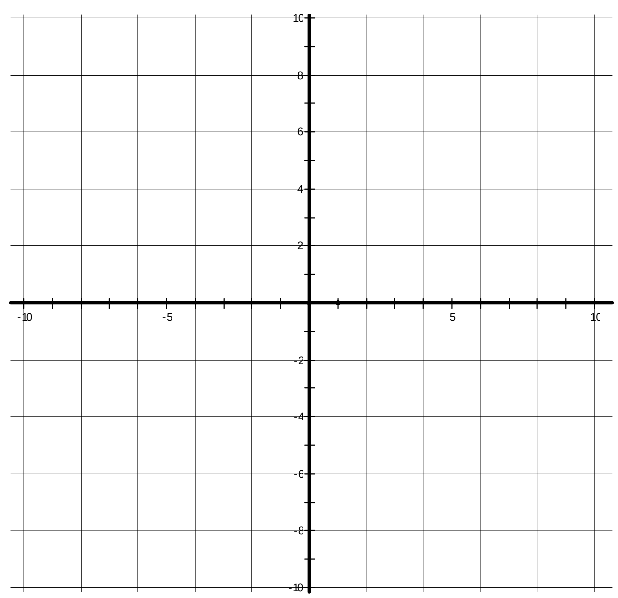
d. $y = 3x^2 + 2x - 3$



The value of *a* is changed, but *b* and *c* are left constant. What effect does this have on the graph?

What does the value of *a* tell you about the parabola?

Example 2: Graph the following quadratic functions on the same grid:



a. $y = x^2 + 3x + 1$

b. $y = x^2 + 2x + 1$

c. $y = x^2 - 5x + 1$

d. $y = x^2 - 4x + 1$

The value of *b* is changed, but *a* and *c* are left constant. What effect does this have on the graph?

What does the value of *b* tell you about the parabola?

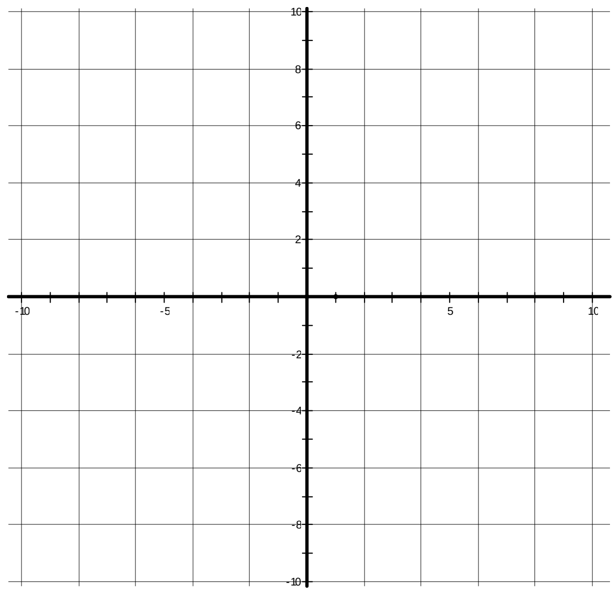
Example 3: Graph the following quadratic functions on the same grid:

a. $y = 2x^2 - 4x + 1$

b. $y = 2x^2 - 4x + 3$

c. $y = 2x^2 - 4x - 2$

d. $y = 2x^2 - 4x$

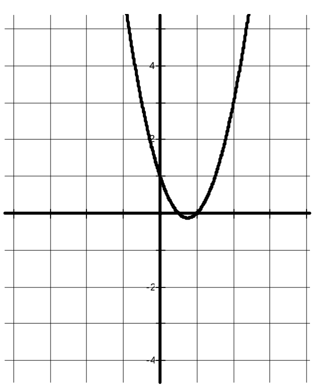


The value of *c* is changed, but *a* and *b* are left constant. What effect does this have on the graph?

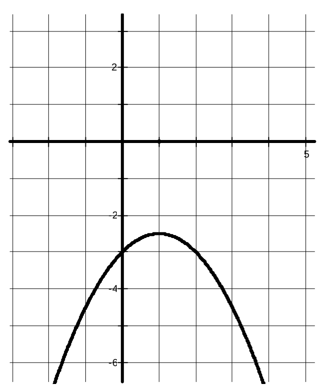
What does the value of *c* tell you about the parabola?

Example 4: The graphs of three quadratic relations are shown. Predict possible values of *a*, *b*, and *c* in the equation for each graph.

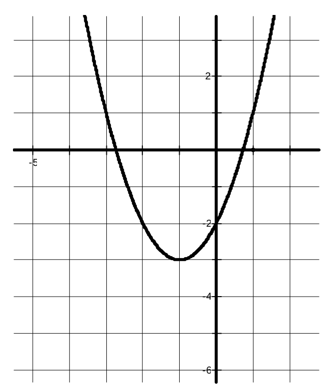
a.



b.



c.



Assignment: pg. 360 #1-6

7.2 Properties of Graphs Of Quadratic Functions

Remember that the **vertex** of a quadratic function is the maximum or minimum point of the parabola. To find the vertex from an equation we can use the symmetry of the parabola to help us.

Example 1: Find the vertex of the following quadratic functions, and state if the vertex is a maximum or a minimum.

a. $y = x^2 - 2x + 4$

x	y

b. $y = -x^2 + 3x - 4$

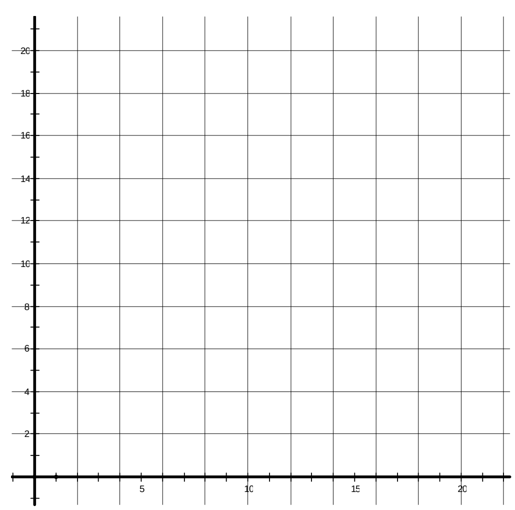
x	y

c. $y = 2x^2 + 6x$

x	y

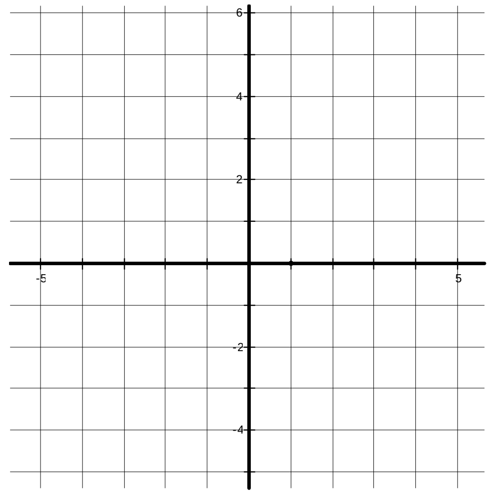
Example 2: A water arch at a splash pad is defined by the quadratic function: $f(x) = -0.15x^2 + 3x$.

Graph the function, and state its domain and range.



Example 3: Sketch the graph of the given function, then find the desired properties:

a. $y = -x^2 - 2x + 3$



i. Vertex: __________

ii. Y-intercept: __________

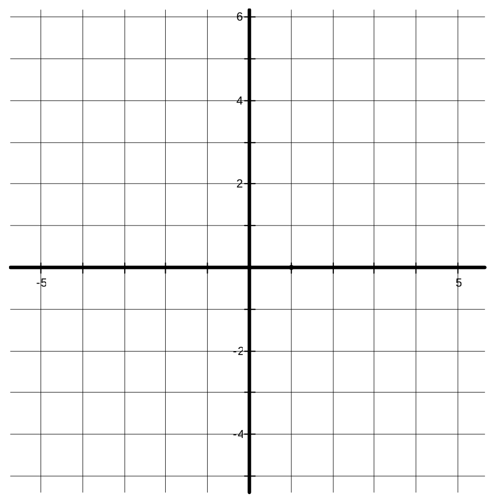
iii. X-intercept: __________

iv. Axis of symmetry: __________

v. Domain: __________

vi. Range: __________

b. $y = x^2 + x - 2$



i. Vertex: __________

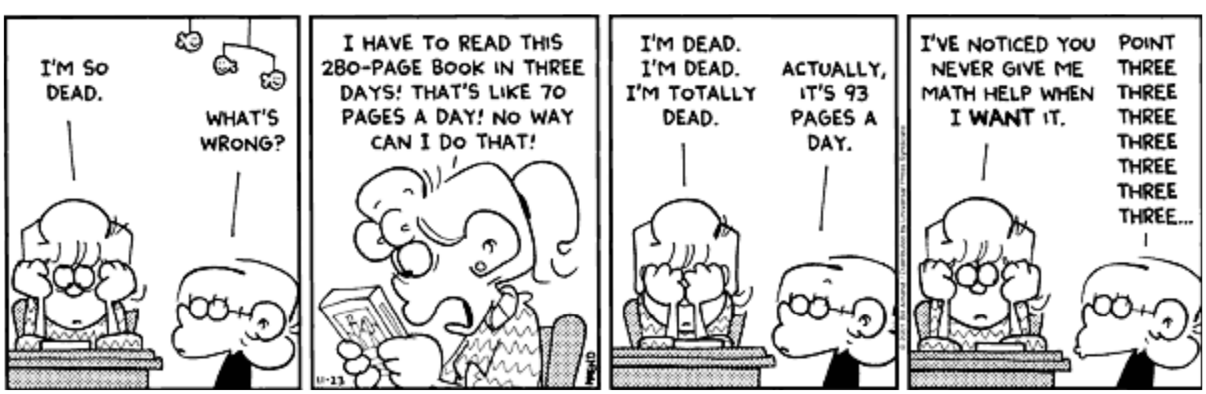
ii. Y-intercept: __________

iii. X-intercept: __________

iv. Axis of symmetry: __________

v. Domain: __________

vi. Range: __________



Example 4: Some boaters use red aerial miniflares in an emergency. The path of one brand of flare, when fired at an angle of 70° to the horizontal, is modeled by the function:

$$h(t) = -9(t-3)^2 + 83$$

where *h(t)* is the height in metres and *t* is the time in seconds since the flare was fired.

a. Sketch a graph of the function, including your window.

b. What is the maximum height of the flare?

c. How many seconds until the flare hits the water?

d. If the flare burns red for 2 seconds, how high is it when it burns out?

Assignment: pg. 368 #1, 4-7, 9-11, 12-14

FOM11

7.3 Solving Quadratic Equations by Graphing

A **quadratic function** is of the form $y = ax^2 + bx + c$. To solve a quadratic function means to find the *x*-intercepts, called the **zeroes.**

A **quadratic equation** is of the form $ax^2 + bx + c = 0$. To solve a quadratic equations means to find the *x*-intercepts, called the **roots.**

Example 1: Solve $3x^2 - 11x - 4 = 0$ using graphing. (ie. Find the roots.)

Example 2: Find the zeroes for $y = x^2 - x - 20$.

Example 3: What are the roots for $x^2 - 6x + 9 = 0$?

Example 4: The manager at Suzie's Fashion Store has determined that the function $R(x) = 600 - 6x^2$ models the expected weekly revenue, *R*, in dollars, from sweatshirts as the price changes, where *x* is the change in price, in dollars. What price increase or decrease will result in no revenue?

Example 5: Solve $3x^2 - 6x + 5 = 2x(4 - x)$ by graphing.

Example 6: Lamont runs a boarding kennel for dogs. He wants to construct a rectangular play space for the dogs, using 40 m of fencing and an existing wall as one side of the play space.

a. Write a function that describes the area, *A*, of the play space in terms of any width, *w*.

b. Determine the number of possible widths for an area of:

i. $250\ m^2$

ii. $200\ m^2$

iii. $150\ m^2$

Assignment: pg. 379 #5-7, 10-12

FOM 11 **Factoring Review**

1. Factor the following expressions:

a) $4x^2 - 14$ **b)** $2x^2 + x$ **c)** $3x^3 - 12x$

2. Factor each trinomial.

a) $x^2 + 4x - 21$ **b)** $x^2 + 7x + 10$

c) $2x^2 - 7x + 6$ **d)** $4x^2 + 11x - 3$

3. Factor each expression fully.

a) $-3x^2 + 9x - 6$ **b)** $2x^2 - 12x + 18$

c) $x^2 - 16$ **d)** $4x^2 - 9$ **e)** $-4x^2 + 49$

Many quadratic equations can be solved by factoring. The *zero product property* states that if *ab* = 0, then *a* = 0 or *b* = 0, or both. Therefore, the roots of a quadratic equation occur when the product of the factors is equal to zero.

Example 4: Solve each equation, then verify the solution.

a. $x^2 - 2x - 8 = 0$

b. $3x^2 - 2x - 8 = 0$

c. $2x^2 + 18 = 12x$

d. $2x^2 = 4x$

Example 5: A football is kicked vertically. The approximate height of the football, h metres, after t seconds is modeled by the formula: $h = 1 + 20t - 5t^2$.

a. Determine the height of the football after 2 s.

b. When is the football 16 m high?

Assignment: Factoring Review Worksheet

FOM 11 **7.4 Factored Form of a Quadratic Function**

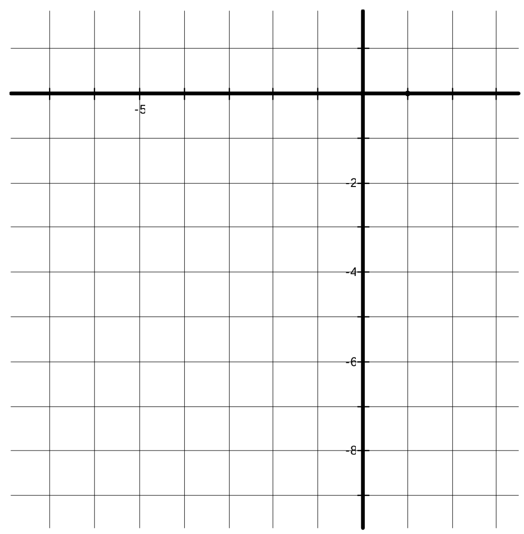
Remember, any quadratic equation can be written in the standard form of a quadratic $ax^2+bx+c=0$ where $a \neq 0$. If this factors easily, we can use the **zero product theorem** to extract the roots (ie. x-intercepts).

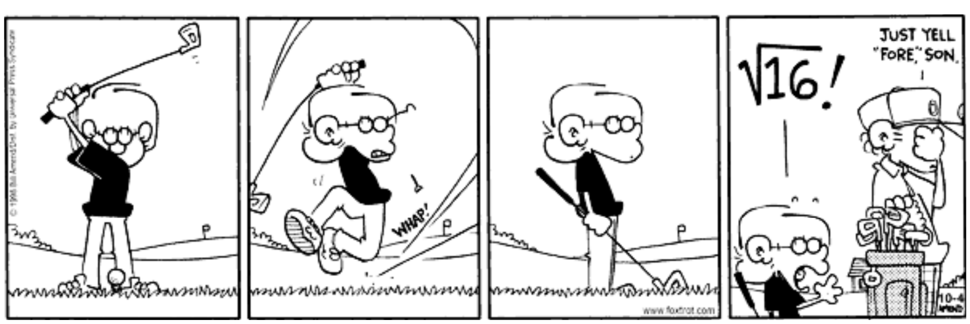
Zero Product Theorem: If $a \times b = 0$, then $a=0$ or $b=0$.

Eg. $(x+5)(x-2)=0$ means $x+5=0 \Rightarrow x=-5$ or $x-2=0 \Rightarrow x=2$.

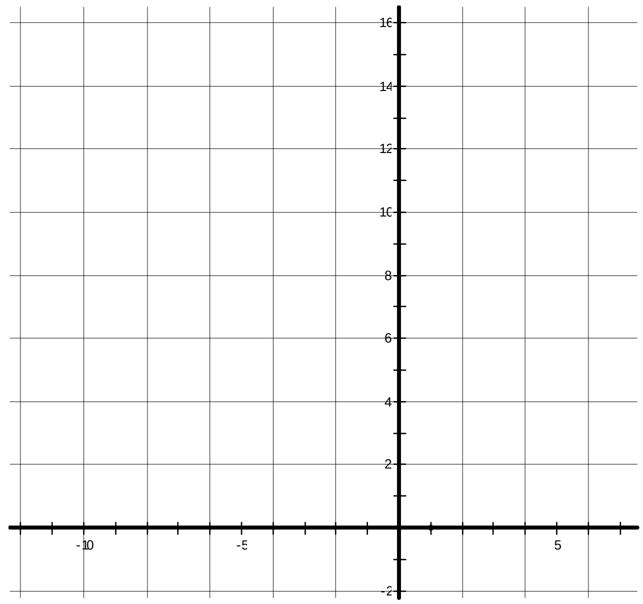
We can use factoring or partial factoring to help us sketch a quadratic function given in standard form.

Example 1: Sketch the graph of $y=2x^2+12x+10$ and state the domain and range of the function.



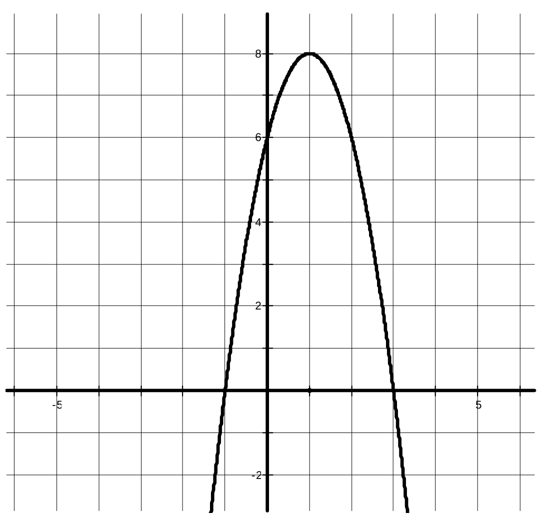


Example 2: Sketch the graph of $f(x) = -x^2 - 3x + 12$ and state the domain and range.

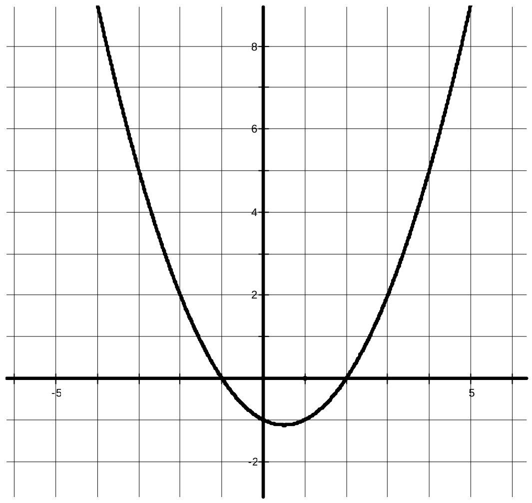


Example 3: Determine the equation of the function that defines each graph. Write each function in standard form.

a.



b.



Example 4: A career and technology class at a high school in British Columbia operates a small T-shirt business out of the school. Over the last few years, the shop has had monthly sales of 300 T-shirts at a price of \$15 per T-shirt. The students have learned that for every \$2 increase in price, they will sell 20 fewer T-shirts each month. What should they charge for their T-shirts to maximize their monthly revenue?

Assignment: pg. 391 #2, 3, 8, 9, 10ace, 11, 16

FOM 11 **7.5 Solving Quadratic Equations by Factoring**

We have learned how to solve a quadratic equation by graphing. Our next method of solving a quadratic equation is by **factoring**.

Example 1: Solve the following quadratic equations:

a. $x^2 - x - 6 = 0$

b. $2x^2 + 6x = 0$

c. $6x^2 - 7x = 5$

d. $x(3x + 1) = 2$

Example 2: A rectangular garden has dimensions 5 m by 7 m. When both dimensions are increased by the same length, the area of the garden increases by 45 m^2. Determine the dimensions of the larger garden.

Example 3: A football is kicked vertically. The approximate height of the football, h metres, after t seconds is modeled by the formula: $h = 1 + 20t - 5t^2$.

a. Determine the height of the football after 2 s.

b. When is the football 16 m high?

When $b = 0$, the quadratic equation $ax^2 + bx + c = 0$ becomes $ax^2 + c = 0$. If this equation has a solution, it can be solved by using square roots.

Example 4: Solve each equation and verify the solution.

a. $3x^2 - 7 = 8$

b. $(x+3)^2 = 20$

Example 5: Write an equation of a quadratic function with zeroes $\frac{2}{3}$ and $-\frac{1}{2}$. Is the equation you found the only equation possible?

Example 6: Matthew solved a quadratic equation as shown. Identify and correct any errors in his solution.

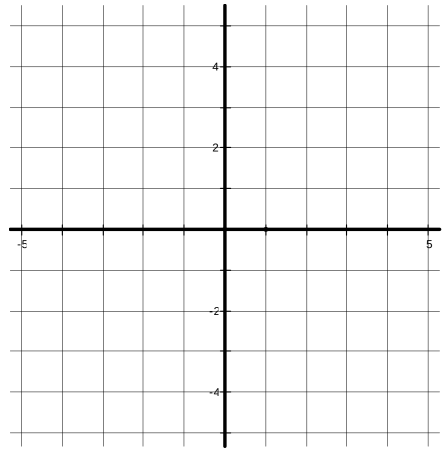
$$4x^2 = 9x$$
$$\frac{4x^2}{x} = \frac{9x}{x}$$
$$4x = 9$$
$$x = 2.25$$

Assignment: pg. 405 #1, 2, 5-11, 13, 14

FOM 11 **7.6 Vertex Form of a Quadratic Function**

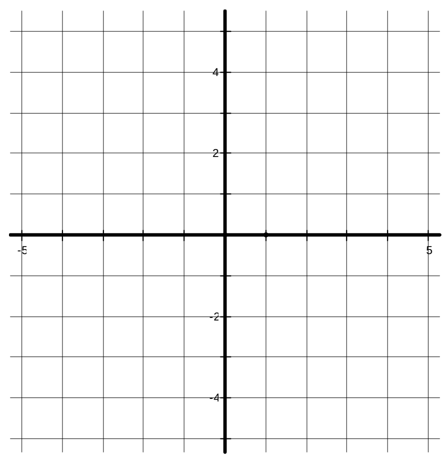
The graph of $y = x^2$ is a parabola with the following properties:

i. vertex ______________________
ii. y-intercept ______________________
iii. x-intercept ______________________
iv. opening ______________________
v. axis of symmetry ______________________
vi. domain ______________________
vii. range ______________________



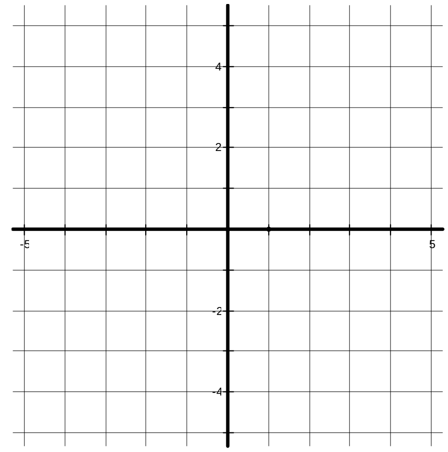
The graph of $y = x^2 + q$ is a parabola with the following properties:

i. vertex ______________________
ii. y-intercept ______________________
iii. x-intercept ______________________
iv. opening ______________________
v. axis of symmetry ______________________
vi. domain ______________________
vii. range ______________________



The graph of $y = x^2$ is simply moved up or down by the value of q.

The graph of $y = (x - p)^2$ is a parabola with the following properties:



i. vertex ____________________

ii. y-intercept ____________________

iii. x-intercept ____________________

iv. opening ____________________

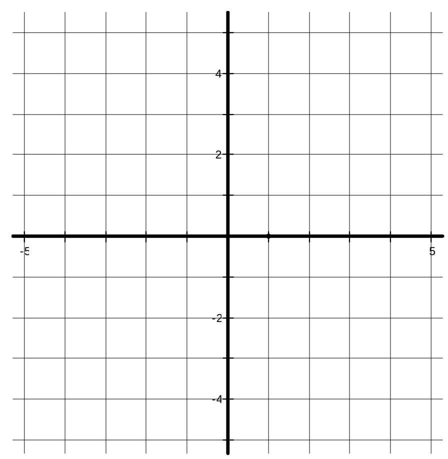
v. axis of symmetry ____________________

vi. domain ____________________

vii. range ____________________

The graph of $y = x^2$ is simply translated left or right by the value of *p*.

The graph of $y = ax^2$ is a graph with the following properties:



i. vertex ____________________

ii. y-intercept ____________________

iii. x-intercept ____________________

iv. opening ____________________

v. axis of symmetry ____________________

vi. domain ____________________

vii. range ____________________

The graph of $y = x^2$ is simply expanded (a>1) or compressed (0<a<1) vertically.

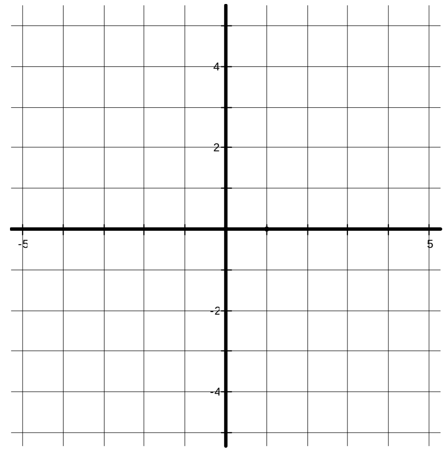
Example 1: Graph:

a. ____________________

b. ____________________

c. ____________________

d. ____________________

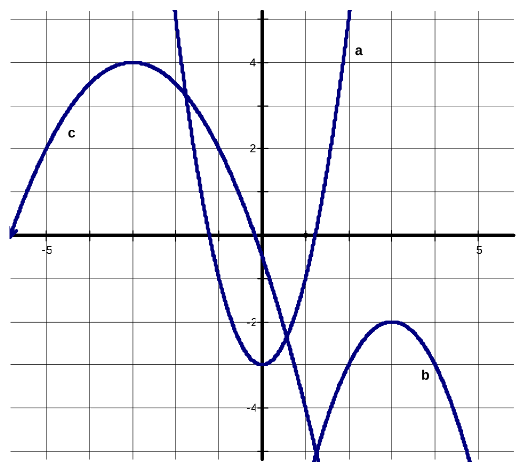


Example 2: Write an equation that could represent each graph:

a. ____________________

b. ____________________

c. ____________________



Example 3: Define three different quadratic functions, in vertex form, that open downward. One function should have two zeroes, another should have one zero, and the third should have no zeroes.

Example 4: A goalkeeper kicked a soccer ball from the ground. It reached its maximum height of 24.2 m after 2.2 s. The ball was in the air for 4.4 s.

a. Define the quadratic function that models the height of the ball above the ground.

b. After 4 s, how high was the ball above the ground?

Assignment: pg. 417 #1-3, 7, 10-12, 15

FOM 11 **7.7 Solving Quadratic Equations Using the Quadratic Formula**

To solve a quadratic equation we can use:

1.__________________________

2.__________________________

3. __________________________

When a quadratic equation, $ax^2 + bx + c = 0$, cannot be factored, a formula can be used to solve for the roots.

> Quadratic Formula:
>
> The solution to $ax^2 + bx + c = 0$ is given by:
>
> $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 1: Solve each equation:

a. $6x^2 - 3 = 7x$

b. $12x^2 - 47x + 45 = 0$

Example 2: Solve each equation:

a. $2x^2 + 8x - 5 = 0$

b. $5x^2 - 10x + 3 = 0$

Example 3: A store rents an average of 750 video games each month at the current rate of \$4.50. The owners of the store want to raise the rental rate to increase the revenue to \$6500 per month. However, for every \$1 increase, they know they will rent 30 fewer games each month. The following function relates the price increase, *p*, to the revenue, *r*:

$$(4.5 + p)(750 - 30p) = r$$

Can the owners increase the rental rate enough to generate revenue of \$6500 per month?

Assignment: pg. 427 #4, 6-8, 10, 11

7.8 Solving Problems Using Quadratic Models

Example 1: The engineers who designed the Coal River Bridge on the Alaska Highway in BC used a supporting arch with twin metal arcs. The function that describes the arch is: $h(x) = -0.005061x^2 + 0.499015x$

where *h(x)* is the height, in metres, of the arch above the ice at any distance, *x*, in metres, from one end of the bridge.

a. Determine the distance between the bases of the arches.

b. Determine the maximum height of the arch, to the nearest tenth.

Example 2: Find two consecutive odd whole numbers such that the sum of their squares is 130.

Example 3: Two planes travel at right angles to each other after leaving an airport at the same time. One hour later, they are 390 km apart. If one plane travels 210 km/h faster than the other, what is the speed of the slower plane?

Example 4: The cold water tap can fill a tub one minute faster than the hot water tap. The two taps together can fill the tub in 4 minutes. How long does it take each tap to fill the tub on its own?

	Time	Fraction of work done in 1 min.	Fraction of work done in 4 min.
Cold Tap			
Hot Tap			
Together			

Example 5: The length and width of a rectangular sheet of paper is 8 in by 11 in. How much must be added equally to the length and width to double the area?

Assignment: pg. 436 # 3-9